Math 2233.003
SAMPLE SECOND EXAM – Summer 2017

Name:_____

1. Given that $y_1(x) = x^{-1}$ and $y_2(x) = x^2$ are solutions to $x^2 y'' - 2y = 0$

(a) (5 pts) Show that the functions $y_1(x)$ and $y_2(x)$ are linearly independent.

(b) (5 pts) Write down the general solution.

(c) (5 pts) Find the solution satisfying the initial conditions $y(1) = 2$, $y'(1) = 1$.

2. (10 pts) Given that $y_1(x) = x^{-2}$ is one solution of $x^2 y'' + 5xy' + 4y = 0$, use Reduction of Order to determine the general solution.

3. (10 pts) Explain in words and formulas how you would construct the general solution of $y'' + p(x) y' + q(x) y = g(x)$, given that $y_1(x)$ is a solution of $y'' + p(x) y' + q(x) y = 0$. (That is, describe the general procedure, writing down the relevant formulas. It is **not** necessary to carry out any calculations.)

4. Determine the general solution of the following differential equations.

(a) (5 pts) $y'' - 3y' - y = 0$

(b) (5 pts) $4y'' - 4y' + y = 0$

(c) (5 pts) $y'' + 4y' + 13y = 0$

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(d) (5 pts) $x^2 y'' + 3xy' - 8y = 0$

(e) (5 pts) $x^2 y'' - 3xy' + 4y = 0$

(f) (5 pts) $x^2 y'' - 2xy' + 3y = 0$

5. Given that $y_1(x) = e^x$ and $y_2(x) = e^{-3x}$ are solutions of $y'' + 2y' - 3y = 0$.

(a) (10 pts) Use the Method of Variation of Parameters to find a particular solution of $y'' + 2y' - 3y = e^{2x}$.

(b) (5 pts) Find the solution of the differential equation in part (a) satisfying $y(0) = 0$, $y'(0) = 2$.